NOTE

Using Counterintuitive Problems in Teaching Physics

Juan Miguel Campanario, Grupo de Investigación en Aprendizaje de las Ciencias, Departamento de Física, Universidad de Alcalá, 28871 Alcalá de Henares, Madrid, Spain; fscampanario@alcala.es

In the teaching and learning of science, problem-solving exercises are part of the canon of traditional methodology. Students frequently criticize these exercises as nothing more than a "joke," with numeric data often presented in the same order in which they fit into the "proper" formula. A little substitution, and the problem is "solved." As a result, students develop a careless methodology, looking for the short response that will fill the blanks of a mathematical exercise and require no critical thinking. Moreover, they are haunted by previously learned "intuitive physics"—full of misconceptions used to understand the world around them—that must now coexist with "academic physics" to pass exams. The educational goal of relating daily phenomena and scientific concepts may never be achieved.

There is a vast literature outlining new approaches to helping students solve traditional problems. One alternative proposes solving problems as if they were small research projects, disregarding the use of numerial values in problems and thereby putting the emphasis on physics rather than on mathematics. Of course there are many situations for which traditional problems with their numeric data are still useful and cannot just simply vanish from physics teaching. Computation is a key element in physics training and students have to know how to handle it.

I suggest the use of in-class problems that yield solutions that challenge students' expectations or are worded in such a manner that students obtain a wrong solution by making some standard mistakes. Problems with counterintuitive solutions force students to think before rushing ahead into calculations. Soon they become aware of discrepencies between their existing ideas and the solution they find for the problem.

There are various sources of traditional physics problems to use with this new approach. I have found some problems in the ordinary textbooks and modified them according to the goals of the method. I have invented others while teaching introductory physics to first-year university students. Although it is not difficult to find or write problems similar to those given here, it does require a little effort and some imagination. Many traditional problems can be reworded to alter the goal of the exercise and obtain one with counterintuitive results. For example, situations in which masses or initial velocities do not influence the results are perfectly suited for counterintuitive problems. Some popular books on recreational physics and books with questions and answers referring to physics are also useful as a source of amusing and interesting material.

Some Sample Problems

The problems given here are centered on mechanics, one of the branches of physics in which a good deal of research has been done on students' misconceptions. Introducing problems with counterintuitive results at the very beginning of the course is potentially quite useful.

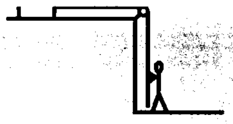
Measurement in Physics

Suppose we take a soccer ball and with the help of a string measure its circumference. Suppose now that we wrap the string one meter away from the surface of the ball. Students rapidly realize that a longer string is necessary to complete that task. We need an additional ΔL meters of string. But let us extrapolate the task and ask the students to measure Earth's circumference with, understandably, a longer string. In continuation (still pretending), they wrap a string around Earth, one meter away from its surface. They will need $\Delta L'$ more meters of string. The challenge is: find a correlation between ΔL and $\Delta L'$.

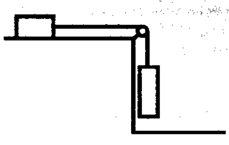
Apparent answer: $\Delta L' >> \Delta L$
Correct answer: $\Delta L' = \Delta L$

Anyone would imagine right away that we will need a much longer string to complete the task of wrapping it at 1 m from the surface of the Earth rather than 1 m from the surface of the soccer ball. However, it is easy to demonstrate that $\Delta L' = \Delta L$. Using this problem in one of the first lectures is fair warning to students that common sense is not always useful for learning physics.

Acceleration



A man pulls a cart on a friction-free surface by means of a pulley and rope as shown in the figure. He pulls with a force F = 600 N. Now, replace the man's action with an iron weight of certain mass that is hooked to the free end of the rope. The cart is set in motion again. Is the acceleration of the cart the same?



[Use g = 10 ms^{-2}, m(cart) = 50 kg, M (weight) = 60 kg.]

Apparent answer: Acceleration is the same because the force acting on the cart is the same.

Correct answer: The cart accelerates less than before.

Students often fail to recognize that from the dynamic point of view the settings are different in the two situations. Students make the mistake of concentrating only on the cart, and consequently assume that the processes are similar because they have not taken into account the mass of the iron block.

Falling Bodies

A man is standing on a balance whose scale indicates a given weight P. Suddenly he squats with acceleration a. What value of weight, P', will the scale indicate while the man is squatting?

Apparent answer: The P' weight is greater because while squatting the man is pushing down on the balance.

Correct answer: The P' weight is lower because while the man is squatting the center of gravity is being accelerated downwards.

A classic problem in almost every textbook involves computing the apparent weight of objects placed on balances in moving elevators. This problem is similar to the "Falling Bodies" problem above. Most new physics students believe that while the man is squatting the scale will reflect a greater weight because he is "pushing down" on the balance. It may be necessary to encourage students to try the experience, because they may be hard to convince otherwise. However, because the squatting person is accelerated downward, the normal force (and thus, reaction or apparent weight) is of smaller magnitude.

Friction Forces

An object of 10-kg mass is laid on a flat surface. A force F = 4 N is exerted on the object at a positive angle of 30° with the horizontal surface. Assume $\mu = 0.5$. Find the resultant force acting on the object. Compare this result with the resultant force of the object when F forms a negative angle of 30° with the horizontal surface.

Apparent answer: Resultant force is directed to (pointing toward) the left.

Correct answer: Resultant force is equal to 0 (friction force is equal to applied force).

Students are so accustomed to careless methodology and mechanical calculations that they obtain wrong solutions to this problem. Given that the problem provides a friction coefficient, many take the wrong step of computing the friction force first. The rule they use states that $F_f = \mu N$, where N is the normal force exerted by the surface on the block. However, N depends both on the weight and the vertical component of F. After doing the calculations, students realize (much to their surprise) that the horizontal component of the net force is directed toward the left. Thus, the net force is always against F and the block will move contrary to the applied force. Who can deny a very counterintuitive result? This example illustrates how students can discover on their own that there is something wrong with their ways of solving problems.

Centripetal Force

A stone with 2 kg of mass tied to a string 1 m in length is subjected to a circular motion within a plane parallel to the floor. The maximum tension that the string can withstand is T = 30 N. What is the maximum reachable velocity of the stone without bringing the string to its breaking point?

Apparent answer: $T = \dfrac{mv^2}{r}$

Correct answer: $T_x = mv^2/r,\ T_y = mg$

This is a very treacherous problem because even the wording is misleading. Obviously an upward force must balance out the weight of the stone, so the angle between a vertical axis and the string has to be less than 90°. Once this is figured out, the apparently easy answer has to be worked over a bit more: the vertical component of tension must balance out the weight. The other component is the centripetal force. Once students discover this, they can proceed with the rest of the computations.

More Falling Bodies

In the movie *Star Trek V*, Captain Kirk falls into a crater that is apparently more than 100 m deep. Acting on impulse, Mr. Spock rushes to the rescue, using a portable rocket-chair, and dives after him. Spock is able to grab his beloved captain a moment before he hits the ground; hence, he saves Kirk's life. Can this be true?

Given that in the movie Captain Kirk is stopped in the last second, he would probably suffer the same consequences because of his sudden deceleration as he would in hitting the ground. In addition, Mr. Spock would have suffered even more serious damage because, in order to overcome Kirk's fall, Spock would need to move downwards with an acceleration greater than *g*. Thus the deceleration he might have been exposed to would necessarily be greater than the deceleration of Captain Kirk.

Comments

In a unique way, integrating exercises with counterintuitive solutions challenges some conceptual errors common among students. The counterintuitive result or the wrong result will point out the inadequate initial analysis and superficial approach. My experience is that when this new approach is used in a systematic way for three or four weeks, students trade in their initial tendency to answer quickly for a deeper analysis of the physics involved in the situation. Soon they try to avoid answering according to their misconceptions and try instead to use correct physics principles and laws. Other physics teachers may find the idea worth trying. I would appreciate feedback concerning this method and suggestions on how it might be improved or restructured.

Acknowledgment

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